

Experimental investigation of rarefied gas flow in different channels

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An experimental investigation of flows of a large number of inert and polyatomic gases in various channels, a non-ideal orifice, flat slits with different surface roughnesses and wall materials, capillary packets with molten walls and a capillary sieve, has been made.

The unsteady flow method and a highly sensitive capacitance micromanometer were used (the sensitivity being $\sim 3 \times 10^{-4} \text{ N/m}^2 \text{ Hz}$). Measurements were made in a range of Knudsen numbers 5×10^4 – 10^{-3} at $\sim 293^\circ \text{K}$, and some measurements for flow through a non-ideal orifice were carried out at 77.2°K .

It was found that, both in the viscous slip-flow and free-molecule regimes for the channels with molten walls, the experimental conductivities were higher (by $\sim 15\%$) than theoretical ones calculated assuming diffuse molecular scattering by the walls. We have also observed that the channel conductivity essentially depends on the channel surface roughness and on the kind of gas. The larger the roughness height, the lower the conductivity. From the experimental data the tangential momentum accommodation coefficients were calculated.

1. Introduction

Analysis of experimental data on rarefied gas flow in channels with artificially rough walls which have appeared in recent years shows that the observed gas volume flow rates are, as a rule, lower than theoretical ones calculated assuming entirely diffuse molecular scattering by the walls (Gaede 1913; Huggel 1952; Lund & Berman 1958, 1966*a, b*; Maegley & Berman 1972). The discrepancy is especially noticeable in the free-molecule regime and flow close to it; the greater the relative mean roughness height, the greater is this discrepancy (Davis, Levenson & Milleron 1964).

The decrease in the observed gas volume flow rate relative to the theoretical value for a smooth wall can be explained by what happens near a rough wall if it is supposed that each element of the roughness scatters molecules only diffusely. Such a problem has been solved by Davis *et al.* (1964) using the Monte-Carlo method. Fairly good agreement between experimental and theoretical results has been obtained. The results (De Marcus 1959; Porodnov *et al.* 1972) for a flat slit also confirm the fact that this decrease in the gas volume flow rate is due to wall roughness.

Experiments on gas flow in capillaries with highly polished or molten walls

show that the observed gas volume flow rates are greater than the theoretical ones (Borisov, Porodnov & Suetin 1972; Davis *et al.* 1964). Such a discrepancy in data, most probably, confirms the idea that among the molecules reflected by a wall there are some whose tangential momentum component is to some extent preserved in the collision. It should be mentioned that such an effect could also be due to surface diffusion of gas molecules. However, experiments on molecular beams interacting with solids (Hinchen & Folley 1965; Hurlbut 1963) confirm the fact that a diffuse/specular scheme of boundary conditions is, apparently, more probable.

Thus, the results of experiments on gas flows in channels with rough walls can be explained even by the simplest scheme of boundary conditions: diffuse molecular scattering by each element of the surface. The diffuse/specular scheme of boundary conditions enables us to describe experiments with gas flows in channels with highly polished or molten walls (Davis *et al.* 1964; Suetin *et al.* 1973).

In the present paper the results of a systematic experimental investigation of the flow of a number of gases through a non-ideal orifice, in capillary packets and through a capillary sieve, as well as through a flat slit with walls of different roughnesses, are given. A comparison of the observed and theoretical results has been made.

2. Experimental technique

The method for unsteady flow first proposed by Knudsen (1909*a, b*) has been used as an experimental method to investigate gas flow in channels. The essence of this method is to measure the relaxation time of the pressure difference ΔP created between two reservoirs V_1 and V_2 which are connected by an experimental channel.

Here we omit the description of the experimental apparatus, as well as the measurement method, as our experimental technique is described elsewhere (Borisov, Porodnov & Suetin 1972; Porodnov, Suetin & Borisov 1970; Suetin *et al.* 1973). We point out only the main details which to some extent are relevant to the correctness of the results obtained.

To register the pressure difference as a function of time we used a highly sensitive micromanometer with a digital reading (Borisov, Kalinin, Porodnov & Suetin 1972), its sensitivity being $\sim 3 \times 10^{-4} \text{ N/m}^2 \text{ Hz}$. Use of the digital micromanometer made it possible for us to measure the time co-ordinate with high accuracy ($\sim 10^{-6} \text{ s}$) at precisely fixed time intervals.

While performing the experiment it is of importance to take into account possible systematic errors. One of them may be due to gas adsorption on the walls of the reservoirs, because the process being investigated is unsteady. This phenomenon can be checked in the following way. From the molecular balance equation for the reservoirs V_1 and V_2 , it is easy to find the dependence of the pressure difference between the channel ends on time:

$$\Delta P = \Delta P_0 \exp(-t/\tau_p), \quad (1)$$

where ΔP_0 is the value of the pressure difference at the instant $t = 0$.

The relaxation time τ_p is defined as

$$\tau_p = \frac{V_0}{M} \left(1 + \frac{\Sigma}{2V_0}\right) \left(1 + \frac{aP}{V_0}\right), \quad V_0 = \frac{V_1 V_2}{V_1 + V_2}, \quad M = \frac{G}{\Delta P}. \quad (2)$$

Here M is the gasdynamic channel conductivity; G is the gas flow rate when the pressure difference between the capillary ends equals ΔP ; a is a characteristic of the elastic membrane of the capacitance manometer and is numerically equal to the change in volume of each reservoir (V_1 and V_2) due to the membrane flexure when $\Delta P = 1\text{N/m}^2$; P is the mean pressure in the system. The quantity Σ allows for gas adsorption on the walls of either chamber and is determined by the mean lifetime τ of molecules in the adsorption state as

$$\Sigma = \frac{1}{4} v_i S \tau, \quad (3)$$

where v_i is the mean thermal velocity of molecules and S is the surface area of one of the chambers.

The change in the pressure difference in the course of the experiment was recorded by a frequency-meter. The reading F_i taken at $t = t_i$ may be written in the form

$$F_i = (F_0 - F_{00}) \{1 - k(F_0 - F_{00}) [1 - \exp(-t_i/\tau_p)]\} \exp(-t_i/\tau_p) + F_{00} + \beta t_i. \quad (4)$$

Here, F_{00} is the frequency measured when there is no pressure difference in the capacitance manometer at $t = 0$, F_0 is the initial value of the frequency (a first reading), β is 'the drift' of frequency-meter readings when $\Delta P = 0$ and k is the nonlinearity coefficient of the micromanometer characteristic. t_i is measured at accurately fixed time intervals Δt , from 2 to 20 s, with accuracy provided by the stability of a quartz generator.

$F_i(t_i)$ was determined by the nonlinear least-squares method for two and three unknown parameters using a computer.

The mean-square error in the relaxation time τ_p was defined in the usual way. It was equal to $0.01 \pm 1.5\%$ and calculated for each function $F_i(t_i)$. A relative measurement error in the reduced system volume V_0 was less than 0.15% . The mean absolute pressure in the working system was determined using a calibrated reservoir with an error of less than 0.2% .

The value of a was determined in three different ways: (a) using a calibration of the micromanometer characteristics at different pressures; (b) from experimental values of gas volume flow rates in the viscous slip-flow regime; (c) from the measured volume change due to the membrane flexure. All the measured a values coincided to within 1.5% , the mean value being $1.02 \times 10^{-9} \text{m}^5/\text{N}$.

A non-ideal orifice, packets consisting of 10 and 100 glass capillaries with molten walls, rectangular channels formed by glass plates with differently treated surfaces and also by fluoro-plastic plates, as well as a glass capillary sieve with molten walls were used as experimental channels. All the characteristics of the experimental channels are given in table 1. For some channels the radius (or the height) was determined both by an optical method and the flow method, while treating experimental data in the viscous slip-flow regime. The calculation

Channel	Wall material	Roughness height, h (μm)	Radius, $R \times 10^3$, or height, $d \times 10^3$ (cm)		Length, $L \pm \Delta L$ (cm)	Width, $b \pm \Delta b$ (cm)
			Optical method	Flow method		
Orifice	Platinum	—	7.46 \pm 0.03	—	$\sim 4.5 \times 10^{-4}$	—
Slit 1	Glass	1-5	8.9 \pm 0.1	8.89 \pm 0.02	0.806 \pm 0.002	0.3900 \pm 0.0005
Slit 2	Glass	0.05	9.1 \pm 0.1	9.08 \pm 0.02	0.805 \pm 0.002	0.3950 \pm 0.0004
Slit 3	Diffusive oil film	—	8.9 \pm 0.1	8.90 \pm 0.02	0.812 \pm 0.003	0.3910 \pm 0.0010
Slit 4	Fluoro-plastic	1-5	7.7 \pm 0.2	7.24 \pm 0.02	0.828 \pm 0.002	0.3940 \pm 0.0004
Packet 1 (10 capillaries)	Glass	—	19.97 \pm 0.05	—	11.986 \pm 0.002	—
Packet 2 (100 capillaries)	Glass	—	5.00 \pm 0.05	4.986 \pm 0.003	2.986 \pm 0.002	—
Capillary sieve (~ 640 capillaries)	Glass	—	13.5 \pm 0.1	—	~ 5.1	—

TABLE 1. Characteristics of the experimental channels.

method is described by Borisov, Porodnov & Suetin (1972) and Suetin *et al.* (1973).

A large number of inert and polyatomic gases were used in our experiments. The purity of the gases used was greater than 99.96% for He, Ne, Ar, Xe and N₂, greater than 99.7% for Kr, H₂ and D₂ and greater than 99.5% for CO₂ and CH₄. All were gases passed through a trap containing liquid nitrogen before the working system was filled. All measurements were made at room temperature. Measurements with the non-ideal orifice were also performed at the temperature of liquid nitrogen.

3. Orifice

Investigation of gas flow through an orifice is interesting for two reasons. First, such an investigation may reveal some experimental systematic error, which may be due to the measurement method. In particular, it might be connected with gas adsorption on the walls of a working chamber when the unsteady gas flow method is used. Second, such an investigation is of importance because of the lack of experimental data in a wide range of Knudsen numbers provided that there is a small pressure difference between the reservoirs. The experimental results available up to now are limited both to a narrow range of Knudsen numbers and to large pressure ratios $\sim 10^3$ (Knudsen 1909*a, b*; Liepmann 1960; Sreekanth 1965; Smetana, Sherrill & Short 1967; Lord, Hurlbut & Willis 1967). The measurement technique used earlier did not provide a chance to study the free-molecule and nearly-free flow regimes in detail. Liepmann's (1960) results for these flow regimes have a rather large deviation ($\sim 15\%$).

In our experiments, an orifice in a thin platinum plate obtained with the help of a laser was used. Its parameters are given in table 1. He, Ne, Ar, Kr, Xe, H₂, N₂ and CO₂ gas flows in the range of Knudsen numbers 10^{-2} – 10^3 at temperatures of ~ 293 °K and 77.2 °K were studied. The maximum ratio of the pressure difference ΔP to the mean pressure P increased from 10^{-4} in the viscous slip-flow regime up to 1 in the free-molecule one.

Figure 1 shows for He, Ne, Ar, and H₂ the dependence of the non-dimensional orifice conductivity M/M_0 (M_0 is the conductivity in the free-molecule limit) upon a parameter δ defined by

$$\delta = \frac{1}{2}\pi^{1/2}/Kn = \frac{1}{2}\pi^{1/2}R/\lambda. \quad (5)$$

Here and elsewhere, R is the radius of the orifice (or capillary); λ is the mean free path of molecules and was calculated from the viscosity coefficient for hard spherical molecules in the highest approximations of Chapman–Enskog theory.

Theoretical values of the orifice conductivity M_0 in the free-molecule limit were calculated using the well-known formula

$$M_0 = \frac{1}{4}v_t SW, \quad (6)$$

where $v_t = (8kT/\pi m)^{1/2}$ is the mean thermal velocity, S is the orifice area and W is the transmission probability of molecules through a non-ideal orifice. W was calculated from Berman's (1965) formula.

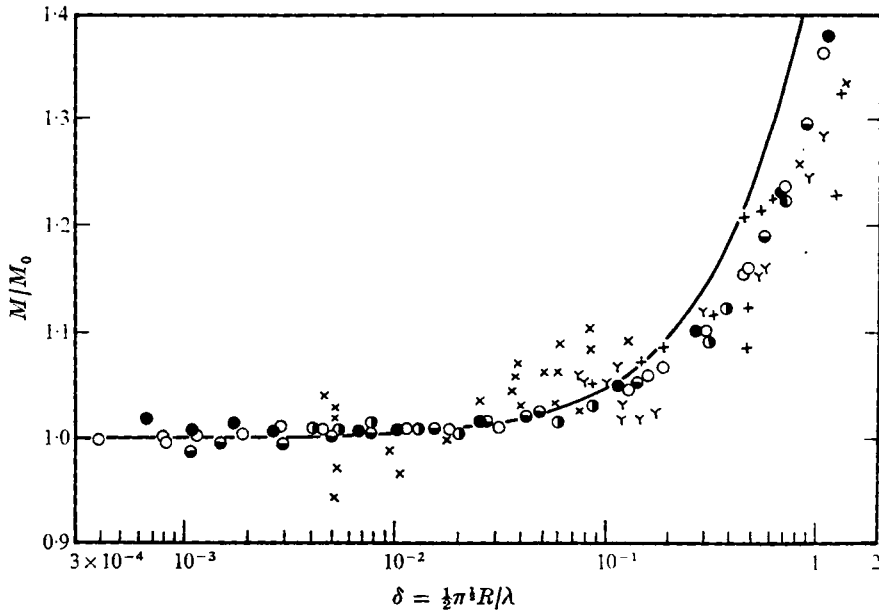


FIGURE 1. Non-dimensional conductivity M/M_0 as a function of the parameter δ (at ~ 293 °K for an orifice in the intermediate and free-molecule flow regimes). —, Willis's (1965) theory. Present data: \circ , He; \bullet , Ne; \bullet , Ar; \ominus , N_2 . Liepmann's (1960) data: \times , He; $+$, Ar; Y , N_2 .

Liepmann's (1960) experimental orifice data, which were obtained with large pressure ratios ($\gtrsim 10^3$), are also shown in figure 1. The solid line in this figure corresponds to Willis's (1965) theoretical relation

$$M/M_0 = 1 + 0.47\delta. \quad (7)$$

For Liepmann's data, the values of δ were calculated for the mean pressures in the system.

Satisfactory agreement of theoretical and experimental results was expected in the range $\delta < 0.1$. When $\delta > 0.1$, the comparison of the results is not valid as Willis's theory is created for a gas flow in vacuum, and only the first iteration, while solving the Boltzmann equation, is used.

The same experimental data plotted with a different scale and range for δ are shown in figure 2. The non-dimensional conductivity M/M_0 for all gases lies on the same straight line and is described by the formula

$$M/M_0 = 0.99(1 + 0.352\delta). \quad (8)$$

The standard relative deviations σ_0 of the observed values of M/M_0 from those calculated from (8) are equal to 0.1, 0.3 and 0.5 % for Ne, Ar and N_2 , respectively. For the rest of the investigated gases σ_0 is less than 0.5–1 %.

The mean value Q_{fm} of M/M_0 in the free-molecule flow regime, with the standard error, is given in table 2 for 293 °K and 77.2 °K. To find Q_{fm} for gases at 293 °K we used the values of M in the range $\delta < 5 \times 10^{-3}$. When Q_{fm} was calculated for 77.2 °K, the values of M used were those in the intermediate and free-molecule

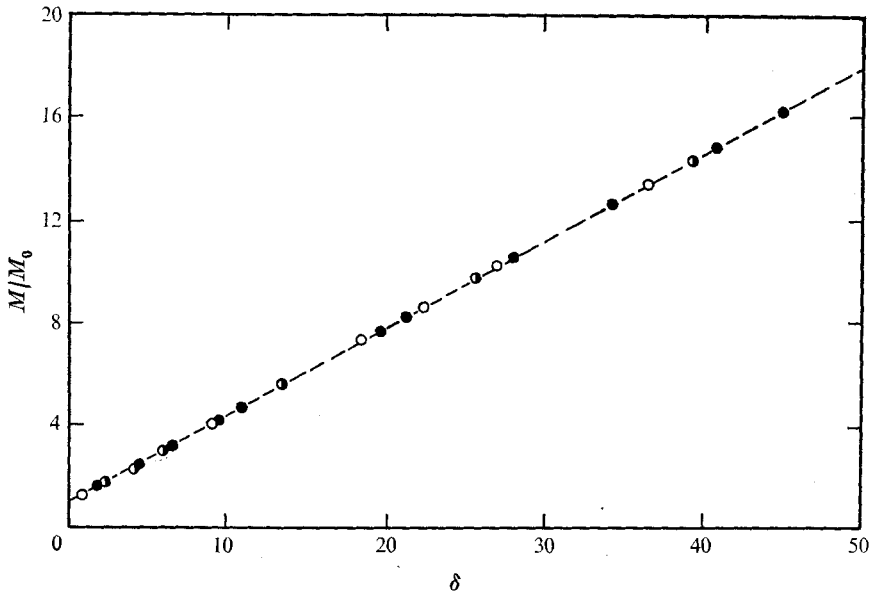


FIGURE 2. Non-dimensional conductivity M/M_0 as a function of the parameter δ (at ~ 293 °K for an orifice in the intermediate and viscous slip-flow regimes). — — —, empirical dependence $M/M_0 = 0.99(1 + 0.352\delta)$; \circ , N_2 ; \bullet , Ne ; \odot , Ar .

Temperature	Gas				
	He	Ne	Ar	H ₂	N ₂
293 °K	1.005 ± 1	1.009 ± 1	1.007 ± 2	1.004 ± 3	1.025 ± 2
77.2 °K	0.993 ± 10	1.032 ± 10	0.992 ± 15	—	0.985 ± 11

TABLE 2. Non-dimensional conductivity $Q_{fm} \pm \Delta Q_{fm} \times 10^3$ in the free-molecule flow regime (for an orifice).

flow regimes ($\delta \lesssim 1$), where experimental results are fairly well described by the formula

$$M/M_0 = Q_{fm}(1 + \beta\delta), \quad (9)$$

with β a constant. In this case the value of Q_{fm} was calculated as a parameter, while the experimental data were treated by the least-squares method.

It seems to us that a comparison of our results with those of others can be made by plotting $(M/P)(\eta/\pi D^3)$, used by Srekanth (1965), as a function of δ , where η is the gas viscosity, D is the orifice diameter and P is the mean pressure in the system. Such a comparison is given in figure 3. This figure also shows the one point of this magnitude which was obtained by Srekanth (1965), Weisberg's (1964) theoretical curve (the broken curve), the semi-empirical curve of Lund & Berman (1966*a*; the solid curve) and the theoretical limits of Wuest (1954) and Roscoe (1949) for the viscous flow regime.

Thus, comparison of the results obtained in the free-molecular flow regime (table 2) with those obtained by extrapolation of experimental data from the

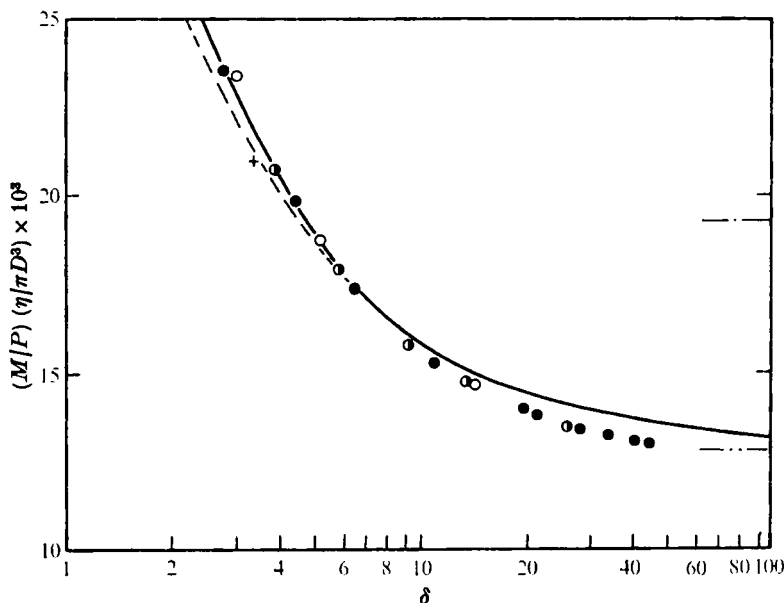


FIGURE 3. Non-dimensional quantity $(M/P)(\eta/\pi D^3) \times 10^3$ as a function of the parameter δ (at ~ 293 °K for an orifice in the intermediate and viscous slip-flow regimes). —, Lund & Berman's (1966) semi-empirical theory; - - -, Weisberg's (1964) theory; - · - ·, Wuest's (1954) limit; · · · ·, Roscoe's (1949) limit. Present data; ○, He; ●, Ne; ●, Ar. +, N_2 , Srekanth's (1965) data.

intermediate flow regime into the range $\delta \ll 1$ with the help of (8) does not reveal any anomalies in the behaviour of M/M_0 for all investigated gases at 293 °K and 77.2 °K.

It should be noted that in the paper by Hegna & Berman (1972) some results on gas flows through a rectangular orifice are reported which greatly differ from ours. For instance, in this paper the experimental curves of M/M_0 vs. δ for He and Ar intersect, and the values of Q_{fm} obtained by extrapolation of experimental results from the viscous slip-flow regime into the free-molecule regime are found to be 1.07 and 1.165 for He and Ar, respectively. Such behaviour of the non-dimensional conductivity M/M_0 seems a little surprising.

4. Rectangular channel (slit)

A flow investigation with inert and some polyatomic gases has been performed for four rectangular slits of similar geometry. Their characteristics are given in table 1. Slit 1 consisted of flat parallel glass plates whose surfaces had irregular roughness. The average height of the surface roughness was $\sim 1-5 \mu\text{m}$. Slit 2 was also constructed using glass plates, whose average roughness height was $\sim 0.05 \mu\text{m}$. Slit 3 was formed from slit 2 by coating its walls with a diffusive oil film. Slit 4 was made out of fluoro-plastic with an average roughness height of $\sim 1-5 \mu\text{m}$. Polished brass foil was used for the slit face planes.

The experimental technique and treatment of the results obtained using slit 2

are described in detail by Porodnov *et al.* (1970). Therefore, here we give only those data which illustrate the influence of wall roughness on the gas flows. All measurements were made at ~ 293 °K.

It is easy to show that in the viscous slip-flow regime the flat slit conductivity is defined by

$$M = a_0 P + b_0, \quad (10)$$

where P is the mean pressure in the system.

If for the gas viscosity coefficient we use the same expression as for hard spherical molecules in the highest approximations of Chapman–Enskog theory, then the coefficients a_0 and b_0 will be given by

$$a_0 = \frac{1}{12\phi} \frac{bd^3}{\eta l_{\text{eff}}}, \quad b_0 = \frac{\pi}{8\phi} \frac{bd^2}{l_{\text{eff}}} v_t \sigma, \quad l_{\text{eff}} = L(1 + \Delta). \quad (11)$$

Here, η is the gas viscosity and σ the slip constant. The quantity ϕ accounts for the fact that the slits being used are of finite width. Δ represents channel end effects due to narrowing (broadening) of the gas stream on entering (leaving) the slit, as well as those due to the presence of technological chamfers near the slit face plane. To evaluate Δ , the well-known Rayleigh method was used; its application to the present problem is considered by Porodnov & Suetin (1971).

Slit heights d and slip constants σ for each of the gases used have been calculated from the experimental data in the viscous slip-flow regime ($\delta > 20$), by the least-squares method using (10) and (11). Average values of the slit heights for all gases are given in table 1 (see the column labelled ‘flow method’). Tabulated values of the gas viscosity coefficients (Golubev & Gnezdilov 1971, chap. 3) were used to calculate d . In all further calculations the values of the slit heights obtained by the flow method were used. The observed slip constants σ are given in table 3. It should be noted that the theoretical value σ_T of the slip constant is equal to 1.147 (Suetin *et al.* 1973). The dependence of the slip constant on the molecular weight for inert gases obtained for slits 1–3 is shown in figure 4.

Comparison of experimental data in the intermediate gas flow regime is difficult because of the lack of a theoretical description of gas flow in a rectangular slit with a small length. Nevertheless, a comparison of the observed data for various gases and slits can be made by introducing the ratio of the experimental slit conductivity M to the theoretical value M_0 in the free-molecule limit, as a function of the parameter δ . To calculate M_0 we used a more exact calculation method than that used by Porodnov *et al.* (1972). This method is based on the application of a variational method to solve a pair of integral equations of the Clausing type (Neudachin, Porodnov & Suetin 1972).

Experimental data for some gases obtained using slit 2 are shown in figure 5. The dependence of the observed values of M/M_0 on δ for He obtained using different slits is given in figure 6.

However, it is worth noting that the values of M_0 used for a rectangular finite slit are not precise. This is due to the fact, as was mentioned above, that our experimental slits had technological chamfers at the slit ends, which probably alter the gas flow conditions in the slit. Hence, a rigorous quantitative comparison

Parameter	Slit	He	Ne	Ar	Kr	Xe	H ₂	D ₂	CO ₂
$\sigma \pm \Delta\sigma \times 10^3$	1	1.24 ± 2	1.25 ± 2	1.31 ± 2	1.23 ± 2	1.33 ± 2	1.22 ± 3	1.27 ± 2	—
	2	1.49 ± 1	1.56 ± 2	1.46 ± 2	1.39 ± 2	1.38 ± 2	1.37 ± 1	1.41 ± 1	1.24 ± 2
	3	1.53 ± 2	1.59 ± 2	1.33 ± 2	—	—	—	1.57 ± 2	—
	4	1.02 ± 3	1.71 ± 2	1.35 ± 2	—	—	1.46 ± 3	1.37 ± 2	—
$Q_{fm} \pm \Delta Q_{fm} \times 10^3$	1	0.899 ± 5	0.925 ± 6	0.881 ± 5	0.835 ± 7	0.816 ± 6	0.850 ± 6	0.870 ± 5	—
	2	0.991 ± 6	1.005 ± 7	0.913 ± 8	0.858 ± 9	0.850 ± 8	0.934 ± 4	0.946 ± 4	0.869 ± 5
	3	1.113 ± 8	1.097 ± 6	0.957 ± 13	—	—	—	1.039 ± 7	—
	4	0.860 ± 6	0.932 ± 11	0.858 ± 11	—	—	0.904 ± 9	0.922 ± 4	—

TABLE 3. Slip constants σ and the non-dimensional conductivity Q_{fm} in the free-molecule flow regime (for the slits).

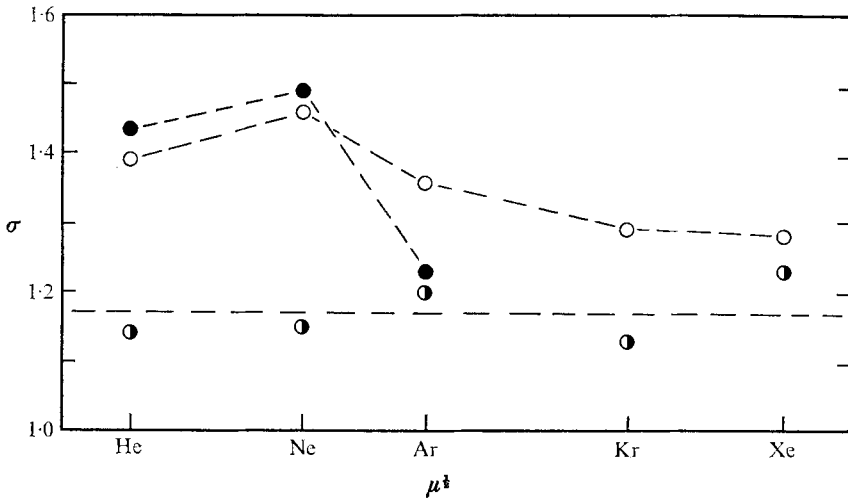


FIGURE 4. Slip constant σ as a function of the molecular weight μ . ●, oil film (slit 3); ○, 'smooth' wall (slit 2); ◐, rough wall (slit 1).

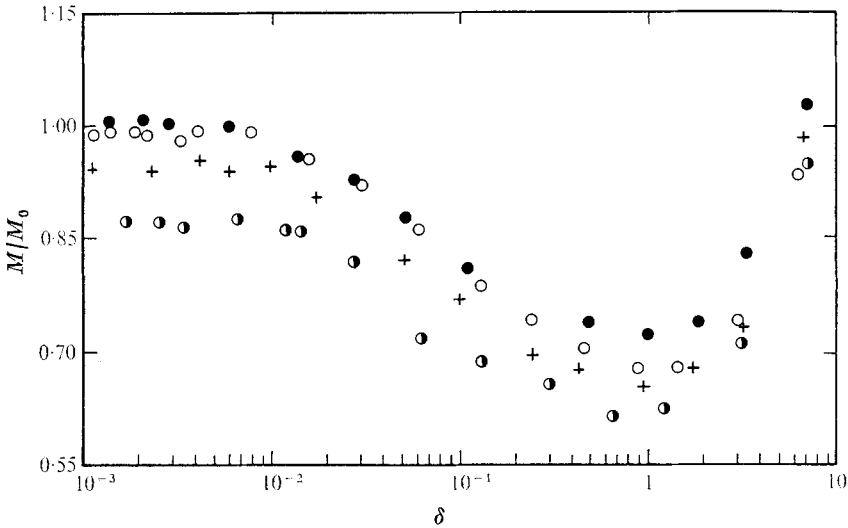


FIGURE 5. Non-dimensional conductivity M/M_0 as a function of the parameter δ (slit 2). +, D₂; ○, He; ●, Ne; ◐, CO₂.

of theory and experiment in the free-molecule flow regime is difficult. However, this circumstance does not prevent us from carrying out the comparison of the results obtained in terms of the ratio M/M_0 , as the geometrical dimensions of the slits are identical.

The free-molecule values Q_{fm} of M/M_0 for all gases and slits investigated are given in table 3. These values were obtained by averaging the M/M_0 values for each gas in the range $\delta < 10^{-3}$, where the conductivity of each slit is constant (figure 5). Note that the theoretical values of the conductivity M_0 of slits 1, 2, 3 and 4 for He are equal to 4.837, 5.127, 4.867 and 3.367 cm³/s, respectively.

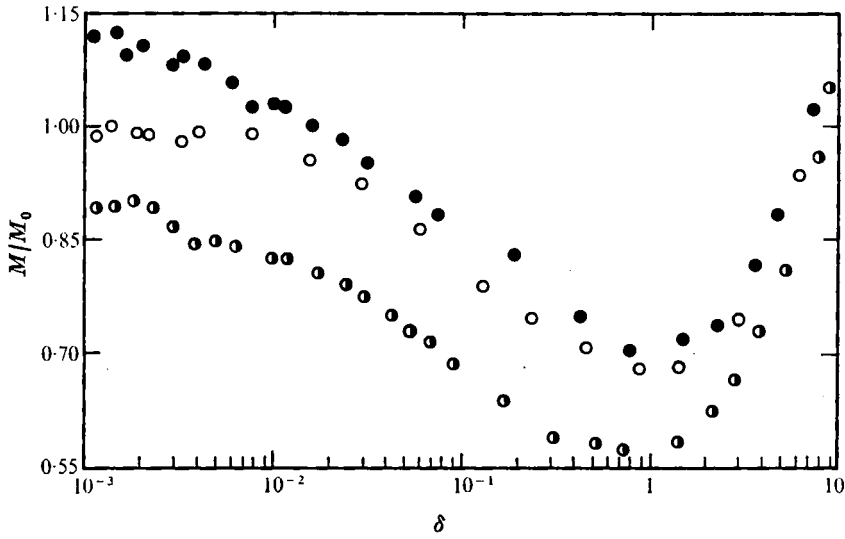


FIGURE 6. Comparison of the experimental results for helium obtained using different slits. ●, oil film (slit 3); ○, 'smooth' wall (slit 2); ●, rough wall (slit 1).

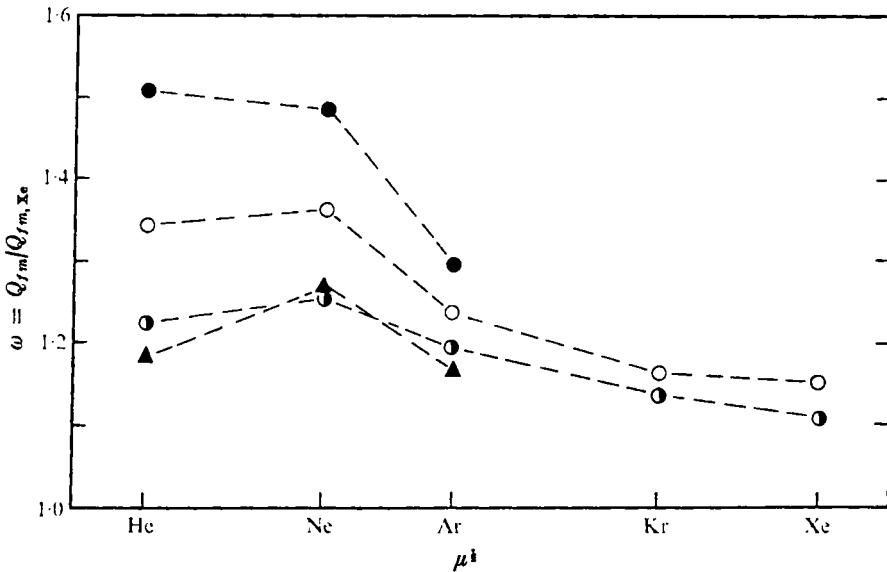


FIGURE 7. Influence of the slit surface roughness upon the flow rates of inert gases in the free-molecule flow regime. ●, oil film; ○, 'smooth' glass surface; ●, rough glass surface; ▲, fluoro-plastic.

The influence of a rectangular slit surface roughness on the size of the gas flow rate in the free-molecule flow regime is illustrated in figure 7, which shows the dependence of ω on the gas molecular weight μ . The variable ω is defined as the ratio of Q_{fm} to $Q_{fm, Xe}$, the least value of the non-dimensional conductivity M/M_0 for Xe, which was obtained using slit 1 (see table 3).

5. Packet of capillaries

As the relaxation time of the pressure difference between the reservoirs for a single narrow capillary is very small, experimental errors can be great; we have therefore used packets consisting of 10 (packet 1) or 100 (packet 2) glass capillaries with molten walls. Geometrical characteristics of the packets are given in table 1. The value of R for packet 2 was found both by the optical method and from the experiments (by the flow method) in the viscous slip-flow regime. The capillaries in packet 1 were calibrated using mercury. The maximum relative deviation of the capillary radii from the mean values equals 0.5 and 2% for packets 1 and 2; that of the capillary lengths is 0.1 and 0.5%, respectively.

The treatment of experimental data obtained using the packets of capillaries is considered by Suetin *et al.* (1973) in detail. Therefore, we describe only the results for the larger number gases investigated here. It should be pointed out that experimental data obtained using packets 1 and 2 coincided within the range of experimental error. All measurements were made at $\sim 293^\circ\text{K}$.

As already mentioned by Suetin *et al.* (1973), if the mean capillary radius is calculated from experimental data using the known viscosity of any (calibrating) gas, then the viscosity coefficients of other gases can be calculated from relative measurements. Argon was used as the calibrating gas; its viscosity was taken from Kestin, Wakeham & Watanabe (1970). The results of the relative measurements of gas viscosity are given in table 4. The viscosity coefficients were adjusted to give the values at 300°K with the help of the formulae corresponding to the model of the repulsion centre. These coefficients are in good agreement with the data of other authors, within 0.5%. This method of determining the viscosity coefficients is described in more detail by Borisov *et al.* (1973).

The rather high accuracy of the conductivity M measurements allowed us to calculate slip flows and the gas slip constants. The dependence of the slip constants for He, Ne and CO_2 on the mean pressure is shown in figure 8. The mean values of the slip constants (table 4) are shown in figure 8 by broken lines. The solid line corresponds to the theoretical value of $\sigma_T = 1.147$ (Suetin *et al.* 1973). This figure illustrates fairly well the fact that the slip constants are independent of pressure, and their systematic divergence for various gases. The dependence of the slip constant upon the gas molecular weight is shown in figure 9.

It is supposed that all differences between experimental and theoretical slip constants occur because of incomplete accommodation of the tangential momentum component of molecules incident on the wall. Following Suetin *et al.* (1973) we have calculated the tangential momentum accommodation coefficient ϵ using the relation

$$\sigma = \sigma_T(2 - \epsilon)/\epsilon, \quad (12)$$

where σ is the experimental slip constant. Experimental values of the accommodation coefficient ϵ are given in table 4.

The conductivity of capillary packets in the intermediate and free-molecule flow regimes is very difficult to measure because of the small gas flow rate and, therefore, large unsystematic errors. That is why investigations in this range of Knudsen numbers were performed only for the inert gases He, Ne and Ar.

Parameter	Gas									
	He	Ne	Ar	Kr	Xe	H ₂	D ₂	N ₂	CO ₂	
σ	1.416	1.504	1.332	1.155	1.120	1.250	1.319	1.345	1.162	
$\Delta\sigma \times 10^3$	9	11	59	45	69	29	13	21	5	
ϵ	0.895	0.865	0.927	0.995	1.010	0.957	0.934	0.925	0.993	
$\Delta\epsilon \times 10^3$	4	4	28	26	40	15	6	14	9	
$\eta ((N s/m^2) \times 10^{-8})$	1972	3203	(2272)	2530	2308	889	1260	1790	1503	
$\Delta\eta ((N s/m^2) \times 10^{-8})$	3	5	7	8	7	4	2	5	1	

TABLE 4. Slip constants σ , the tangential momentum accommodation coefficients ϵ and the viscosity coefficients η in the viscous slip-flow regime (for a packet of capillaries).

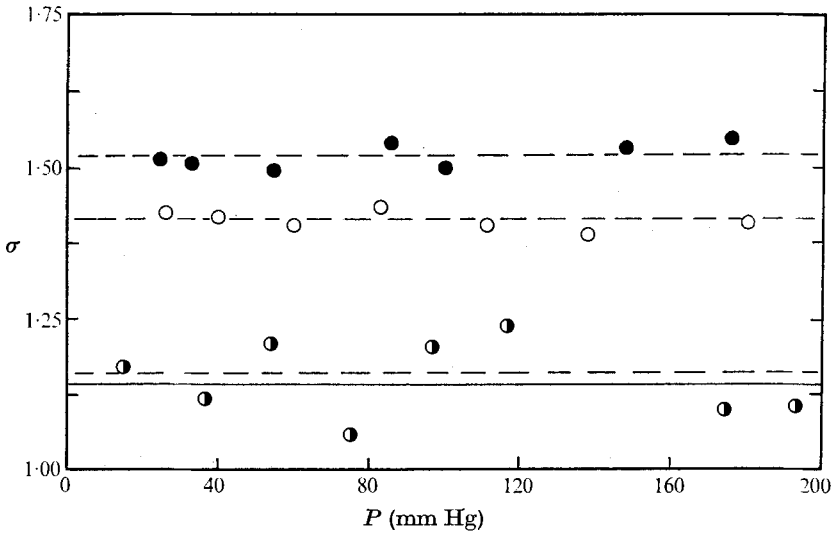


FIGURE 8. Comparison of theory with experiment in the viscous slip-flow regime. \circ , He; \bullet , Ne; \ominus , CO_2 ; —, theoretical slip constant, $\sigma_T = 1.147$; - - -, mean values of the observed slip constants.

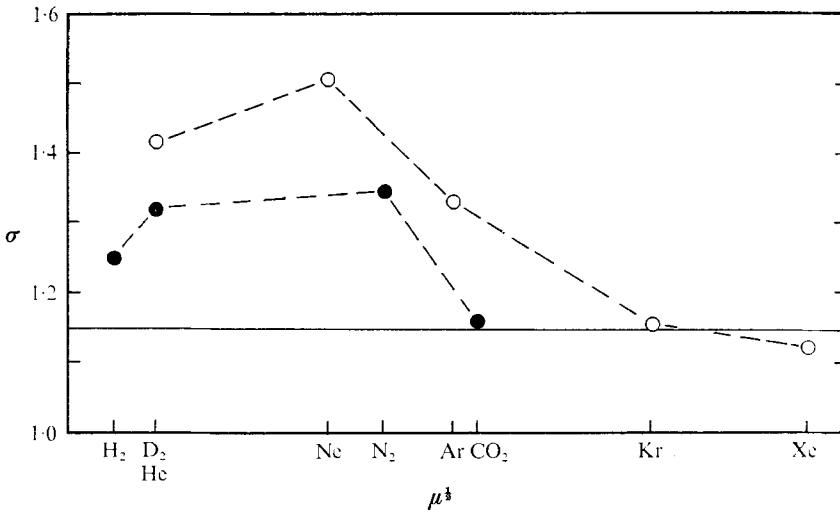


FIGURE 9. Slip constant σ as a function of the molecular weight μ (for a packet of capillaries). —, theoretical slip constant, $\sigma_T = 1.147$; \circ , inert gases; \bullet , polyatomic gases.

In figure 10 experimental values of M/M_0 are plotted *vs.* the parameter δ , where M_0 is the capillary conductivity in the free-molecule flow regime and is defined by the well-known Knudsen formula $M_0 = \frac{2}{3}\pi R^3 v_l/L$. All the values to be inserted in this formula have already been determined. Theoretical curves calculated by Suetin *et al.* (1973) for various tangential momentum accommodation coefficients ϵ are also given in figure 10.

The values Q_{fm} of the non-dimensional conductivity M/M_0 in the free-molecule regime and the corresponding accommodation coefficients ϵ are given in table 5.

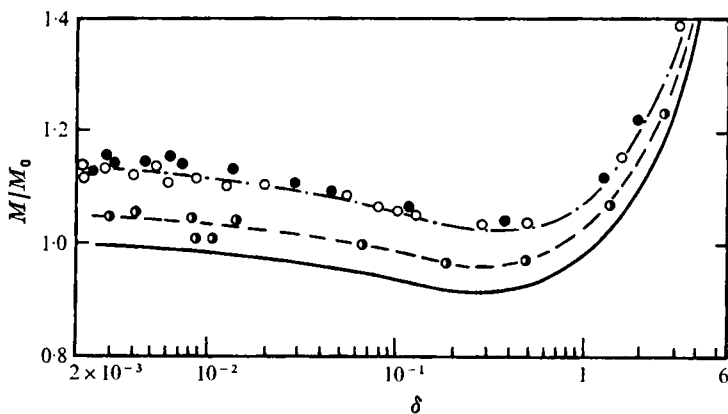


FIGURE 10. Non-dimensional conductivity M/M_0 as a function of the parameter δ (for a packet of capillaries). Theoretical curves (Suetin *et al.* 1973): —, $\epsilon = 1$; - - -, $\epsilon = 0.975$; - · - ·, $\epsilon = 0.935$. Present data: \circ , He; \bullet , Ne; \odot , Ar.

Parameter \ Gas	Gas		
	He	Ne	Ar
$Q_{fm} \pm \Delta Q_{fm} \times 10^3$	1.139 ± 7	1.152 ± 6	1.051 ± 10
$\epsilon \pm \Delta \epsilon \times 10^3$	0.935 ± 4	0.929 ± 3	0.975 ± 6

TABLE 5. Non-dimensional conductivity Q_{fm} and the tangential momentum accommodation coefficients ϵ in the free-molecule flow regime (packet of capillaries).

6. Capillary sieve

A systematic investigation of the intermediate and free-molecule gas flow regimes has been performed using a capillary sieve. This was made from a number of thin-walled capillaries with similar diameter, placed in a glass tube with further sintering and elongation. The sieve's characteristics are given in table 1. The sieve transparency is $\sim 90\%$.

The relative error in the measured sieve conductivity is not more than 0.5% for He, Ne and Ar and 1% for other gases in the whole range of Knudsen numbers (5×10^4 – 0.1). Two independent series of measurements were made. The results of both series coincided to within the limits of experimental error mentioned above.

Unfortunately, there is an uncertainty in the geometry of the capillary sieve (the cross-section of a number of capillaries is not circular), and the exact number of capillaries in the sieve is not known. Therefore, it is convenient to make the comparison of experimental data for various gases in terms of Q , the ratio of the sieve conductivity M to the mean thermal velocity v_t . Such a definition of Q eliminates the dependence of the conductivity M on the kind of gas and the temperature. So all the differences in Q for various gases can be regarded as a result of gas interaction with the surface.

To illustrate this, the reduced sieve conductivity $\gamma = Q/Q_{fm, Xe}$ for He, H₂

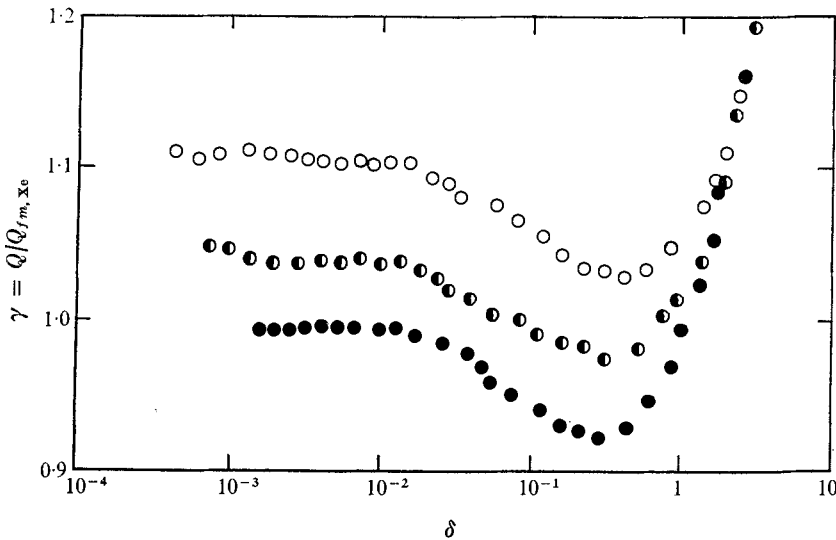


FIGURE 11. $\gamma = Q/Q_{fm, Xe}$ as a function of the parameter δ (for a capillary sieve).
 ○, He; ◐, H₂; ●, Xe.

and Xe is plotted as a function of the parameter δ in figure 11. In the intermediate, free-molecule and nearly-free ($\delta < 0.1$) flow regimes there are essential differences in γ for various gases.

To calculate the free-molecule limit Q_{fm} we used the following empirical formula:

$$Q = Q_{fm}(1 + 0.7\delta \log \delta). \quad (13)$$

The functional dependence of Q on the parameter δ is analogous to the well-known theoretical dependence for the flow regime close to the free-molecular one ($\delta \lesssim 10^{-2}$). The values of Q calculated using (13) coincide with theoretical ones (Suetin *et al.* 1973) to within $\sim 0.2\%$ in the range $\delta < 5 \times 10^{-2}$. All the experimental data in this case were treated by the least-squares method, using (13). The values of $\gamma_{fm} = Q_{fm}/Q_{fm, Xe}$ with the corresponding standard deviations are given in table 6.

The experiments with capillary packets have shown that for gases such as Kr and Xe experimental results practically coincide with theoretical ones both in the viscous slip-flow regime and the intermediate flow regime. This confirms the suggestion that for these gases diffuse scattering of molecules is more probable. Therefore, one can surely assume that Xe molecules are reflected diffusely from the molten capillary walls of a sieve as well. Then, the difference between the value of Q_{fm} for Xe and those for other gases can be explained by a different amount of molecules being scattered diffusely from the wall or by the accommodation coefficient ϵ . This can be calculated from experimental values of Q_{fm} from the equation

$$Q_{fm}(\epsilon) = Q_{fm}(\epsilon = 1)(2 - \epsilon)/\epsilon. \quad (14)$$

According to our assumption, in (14) $Q_{fm}(\epsilon = 1)$ is the reduced conductivity of the sieve for Xe. In this case all the values of ϵ are relative ones. Accommodation

Parameter	Gas									
	He	Ne	Ar	Kr	Xe	H ₂	D ₂	N ₂	CH ₄	CO ₂
γ_{fm}	1.119	1.141	1.037	1.018	1.000	1.054	1.064	1.048	1.016	1.005
$\Delta\gamma_{fm} \times 10^3$	5	5	10	8	11	10	4	11	8	10
ϵ	0.944	0.934	0.982	0.991	1	0.976	0.969	0.977	0.992	0.998
$\Delta\epsilon \times 10^3$	3	3	5	5	5	5	2	6	4	5

TABLE 6. The values of $\gamma_{fm} = Q_{fm}/Q_{fm, \infty}$ and the tangential momentum accommodation coefficients ϵ in the free-molecule flow regime (capillary sieve).

coefficients ϵ are given in table 6. The error in the determination of ϵ is defined only by the error ΔQ_{fm} for each gas as, in this case, the error in the determination of Q_{fm} for Xe is systematic.

7. Concluding remarks

Analysis of these experimental data for various channels with different wall roughnesses allows us to make some conclusions.

(i) Experiments with an almost ideal orifice show that there are no anomalies in the behaviour of the non-dimensional conductivity M/M_0 for all gases in the whole range of Knudsen numbers or the parameter δ (see figures 1 and 2). The free-molecule limits Q_{fm} of M/M_0 for all gases at 293 °K and 77·2 °K obtained both in the free-molecule and nearly-free flow regimes, and by extrapolation of results from the viscous slip-flow regime into the range $\delta \rightarrow 0$, coincide within the limits of experimental error (see table 2).

All this confirms the fact that there are no essential systematic errors due to the unsteady method of measuring gas flow rates.

In the nearly-free flow regime experimental results are satisfactorily described by Willis's (1965) theory only for $\delta < 0.1$. When $\delta \gg 1$, the experimental curve of M/M_0 reaches the theoretical limit obtained by Roscoe (1949) for the viscous flow regime (see figure 3). The fact that there is no exact solution of the problem considered valid in the whole range of Knudsen numbers limits us to using the semi-empirical superposition formula of Lund & Berman (1966*a*). Systematic deviations of the results in this case are not more than 4% (see figure 3).

(ii) The experimental investigation of gas flow in rectangular slits with differently treated wall surfaces has shown that, when the roughness height decreases, the slit conductivity increases for the whole range of Knudsen numbers (see table 2 and figures 4, 6 and 7). The dependence of ω on the kind of gas remains, though the absolute increase in ω with decreasing roughness height is different for all gases and is greatest for He (see figure 7). From the data obtained with glass and fluoro-plastic, one can draw the conclusion that, if the heights of the wall roughness elements are of the same order, the wall material does not essentially influence the gas flow (see figure 7).

We have evaluated the influence of the roughness height and the angle of regular triangular roughness elements on the gas flow in an infinitely wide channel in the free-molecule flow regime by considering diffuse molecular scattering on the roughness surfaces (Porodnov *et al.* 1972). The calculations confirm the fact that, with a fixed channel length-to-height ratio and with a decrease in the relative roughness of the walls from 5% to 0.05%, the gas volume flow rate increases by $\sim 7\%$. In this case of the experiments show an increase in the gas volume flow rate of 9–10% for light gases and 3–4% for heavy gases (see figure 7 and table 3). From figure 7 it is also seen that the gas volume flow rate of He and Xe in a slit with walls coated by an oil film is $\sim 10\%$ more than that in a slit with polished glass walls. It is quite possible that this is due to the molecular smoothness of the oil film, which, most probably, stimulates specular molecular reflexion on the surface.

Thus, the dependence of the gas volume flow rate on the channel surface roughness can be described using only diffuse molecular scattering. However, the dependence of ω on the kind of gas for various slits (see figure 7) suggests that the given model of the boundary conditions is not the only possible one.

(iii) From the experiments on gas flow in capillary packets and a capillary sieve we can say that for glass capillaries with molten walls the macroscopic characteristics of the flow differ essentially from the theoretical ones, this difference depending on the kind of gas (see figures 8–11, and tables 4–6).

Analysis of the experimental results and comparison with theory shows that for heavy and easily condensed gases such as Kr, Xe and CO₂ the tangential momentum accommodation coefficients for molecules on a molten glass surface with high accuracy ($\sim 1\%$) can be considered equal to one, both in the free-molecule and viscous slip-flow regimes (see table 4). The accommodation coefficients for light gases differ greatly from one, their values depending on the flow regime. In particular, in the viscous slip-flow regime the accommodation coefficients are smaller than those in the free-molecule regime (see tables 4 and 5). At present it is rather difficult to explain this fact, but we believe that experiments in a wide temperature range will make it possible to explain this effect.

The coincidence of the measured accommodation coefficients (see tables 5 and 6) justifies the 'relative' method of obtaining accommodation coefficients and the conclusion that $\epsilon = 1$ for Xe.

The observation that for all channels the qualitative dependence of macroscopic characteristics of gas flows upon the molecular weight is similar in any flow regime is of great significance. Probably, this dependence is not affected by the degree of the surface roughness, which makes us think that each roughness element is likely to scatter incident molecules in a diffuse/specular manner. A similar dependence was observed by Lund & Berman (1966*a*).

In conclusion we should like to say the following. The analysis of available experimental and theoretical data shows that the difference between observed and theoretical gas flow rates can be explained by using the diffuse/specular scheme of boundary conditions.

In two recently published studies (Berman & Maegley 1972; Maegley & Berman 1972) the authors have made an attempt to explain experimental data on Ar and He flow between two coaxial cylinders with brass polished walls. This was done with the help of a theory based on the solution of the model kinetic equation (BGK model). They proposed, as boundary conditions, that some of the molecules incident on the surface are diffusely scattered by the surface, while the rest are reflected back in the direction from which they came.

The substitution of real channel walls by smooth walls at which backscattering occurs, proposed by Berman & Maegley, seems to us to be not quite substantiated. Indeed, with such a substitution the backscattering coefficient is, to a great extent, a geometrical but not physical surface characteristic, which makes the correlation between the results of investigations of boundary phenomena by different methods (e.g. by the molecular beam method) more difficult.

REFERENCES

- BERMAN, A. S. 1965 *J. Appl. Phys.* **36**, 3356.
- BERMAN, A. S. & MAEGLEY, W. J. 1972 *Phys. Fluids*, **15**, 772.
- BORISOV, S. F., KALININ, B. A., PORODNOV, B. T. & SUETIN, P. E. 1972 *Pribori i Tech. Exp.* **4**, 209.
- BORISOV, S. F., KALININ, B. A., PORODNOV, B. T. & SUETIN, P. E. 1973 *J. Phys.* **24**, 67.
- BORISOV, S. F., PORODNOV, B. T. & SUETIN, P. E. 1972 *J. Tech. Phys.* **42**, 1310.
- DAVIS, D. H., LEVENSON, L. L. & MILLERON, M. 1964 *J. Appl. Phys.* **35**, 529.
- DE MARCUS, W. C. 1959 *A.E.G. R. & D. Rep. Oak Ridge, Tennessee*, K-1435.
- GAEDE, W. 1913 *Ann. Phys.* **41**, 289.
- GOLUBEV, I. F. & GNEZDILOV, N. E. 1971 *Viscosity of Gases and Gaseous Mixtures*. Moscow: Standartgiz.
- HEGNA, H. A. & BERMAN, A. S. 1972 *Phys. Fluids*, **15**, 1850.
- HINCHEN, J. J. & FOLLEY, W. M. 1965 In *Rarefied Gas Dynamics*, vol. 2, p. 565. Academic.
- HUGGEL, J. A. W. 1952 *Proc. Roy. Soc. A* **212**, 123.
- HURLBUT, F. 1963 *Dynamics of Manned Lifting Planetary Entry*, p. 754. Wiley.
- KESTIN, J., WAKEHAM, W. & WATANABE, K. 1970 *J. Chem. Phys.* **53**, 3773.
- KNUDSEN, M. 1909a *Ann. Phys.* **28**, 75.
- KNUDSEN, M. 1909b *Ann. Phys.* **28**, 299.
- LIEPMANN, H. W. 1960 *J. Fluid Mech.* **10**, 65.
- LORD, R. G., HURLBUT, F. C. & WILLIS, D. R. 1967 In *Rarefied Gas Dynamics*, vol. 2, p. 1235. Academic.
- LUND, L. M. & BERMAN, A. S. 1958 *J. Chem. Phys.* **28**, 363.
- LUND, L. M. & BERMAN, A. S. 1966a *J. Appl. Phys.* **37**, 2489.
- LUND, L. M. & BERMAN, A. S. 1966b *J. Appl. Phys.* **37**, 2496.
- MAEGLEY, W. J. & BERMAN, A. S. 1972 *Phys. Fluids*, **15**, 780.
- NEUDACHIN, I. G., PORODNOV, B. T. & SUETIN, P. E. 1972 *J. Tech. Phys.* **42**, 1306.
- PORODNOV, B. T. & SUETIN, P. E. 1971 *Atomic and Molecular Physics, Proc. Urals Polytech. Inst., Sverdlovsk*, paper no. 189, p. 82.
- PORODNOV, B. T., SUETIN, P. E. & BORISOV, S. F. 1970 *J. Tech. Phys.* **40**, 2383.
- PORODNOV, B. T., SUETIN, P. E., BORISOV, S. F. & NEVOLIN, M. V. 1972 *Izv. Vuzov. Fiz.* **10**, 150.
- ROSCOE, R. 1949 *Phil. Mag.* **40** (7), 338.
- SMETANA, F. O., SHERRILL, W. A. & SHORT, D. R. 1967 In *Rarefied Gas Dynamics*, vol. 2, p. 1243. Academic.
- SREEKANTH, A. K. 1965 In *Rarefied Gas Dynamics*, vol. 1, p. 621. Academic.
- SUETIN, P. E., PORODNOV, B. T., CHERNJAK, V. G. & BORISOV, S. F. 1973 *J. Fluid Mech.* **60**, 581.
- WEISBERG, H. L. 1964 *J. Appl. Phys.* **35**, 142.
- WILLIS, D. R. 1965 *J. Fluid Mech.* **21**, 21.
- WUEST, V. W. 1954 *Ing. Arch.* **22**, 357.